

## NONPERTURBATIVE PHYSICS AT SHORT DISTANCES

V.I. ZAKHAROV

*Istituto Nazionale di Fisica Nucleare - Sezione di Pisa, Largo Pontecorvo, 3, 56127  
Pisa, Italy;*

*Max-Planck Institut für Physik, Föhringer Ring 6, 80805, München, Germany*

There is accumulating evidence in lattice QCD that attempts to locate confining fields in vacuum configurations bring results explicitly depending on the lattice spacing (that is, ultraviolet cut off). Generically, one deals with low-dimensional vacuum defects which occupy a vanishing fraction of the total four-dimensional space. We review briefly existing data on the vacuum defects and their significance for confinement and other non-perturbative phenomena. We introduce the notion of ‘quantum numbers’ of the defects and draw an analogy, rather formal one, to developments which took place about 50 years ago and were triggered by creation of the Sakata model.

*Keywords:* Confinement, topological excitations, Sakata model

### 1. Introduction

This mini-review was prepared for the Conference celebrating 50 years of the Sakata model<sup>1</sup> held at the University of Nagoya in November 2006. This celebration is an emotional event for me since it happened so that I learnt about the Sakata model before I had exams in field theory. In the winter 1960-1961 L.B. Okun was giving at the Institute of Theoretical and Experimental Physics (ITEP) a course of lectures on the Sakata model and weak interactions. By chance, I attended the lectures. Having no education in field theory did not help much to understand physics but I do remember well the excitement and tense expectations of imminent and decisive developments.

Later, I had ample opportunity to learn more about the model since there was exploding activity at ITEP in that direction. In particular, my theses adviser, I.Yu. Kobzarev was a close collaborator of L.B. Okun. There is no need, however, to go into historical remarks here <sup>a</sup> since there appeared

<sup>a</sup>It is difficult for me to leave the recollections grounds without mentioning the role,

recently recollections of L.B. Okun,<sup>2</sup> to be published in the same Proceedings.

Instead, we will proceed directly to the topic of the talk, that is non-perturbative physics at short distances. The connection to the theme of the Conference, as we see it, is as follows. The Sakata model was a pre-runner of the quark model. And quarks acquired solid theoretical status only when it was realized that they survive at short distances. During last five years or so, evidence has been accumulating that confining fields in QCD are actually hard, i.e. survive at short distances. Hence, there is a chance that non-perturbative fields could be understood in terms of fundamental physics. The analogy to fundamental quarks might look too far fetched or even artificial. The reason to draw parallels to sakatons or quarks is to convey amusement brought by the lattice data and facilitate discussion of the data by the continuum community.

The Sakata model explored the idea that there exist fundamental constituents of matter. Roughly speaking, the messages were:

- There might exist a few fundamental constituents. Earlier, one rather thought in terms of a single fundamental field (if at all)
- Even in the absence of detailed dynamical theory one could check the choice of the fundamentals by selection rules

Later, there was found a simple and still meaningful limit to consider dynamics. Namely:

- Constituents fill in an irreducible representation of a simple group, let it be associated with a broken symmetry.

Trying to build up parallels for the non-perturbative physics we will proceed in the following way:

- review briefly the evidence in favor of vacuum defects in Yang-Mills theories (mostly,  $SU(2)$  and  $SU(3)$  cases) and try to identify basic ones.
- Introduce notion of quantum numbers for non-perturbative structures, like dimensionality of the vacuum defects
- Speculate that the quantum numbers of the defects can be predicted by studying classical solutions of large- $N_c$  theories.

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vision and devotion to physics of I.Ya Pomeranchuk which are beginning and end of all my memories of that time.

## 2. Phenomenology of vacuum defects

### 2.1. Critical exponents

In a pure Yang-Mills theory, everything can be expressed in terms of the running coupling constant. Imagine that measurements are performed with resolution  $a$ . In lattice formulation, an example of such a measurement is a Monte-Carlo generated set of vacuum gauge fields  $\{A_\mu^a(x)\}$ . The lattice spacing  $a$  plays the role of resolution of measurements on the vacuum fields.

The coupling normalized at the lattice spacing,  $g^2(a)$  is then the relevant parameter. Because of asymptotic freedom  $\lim_{a \rightarrow 0} g^2(a) \rightarrow 0$ , and the vacuum fields  $\{A_\mu^a(x)\}$  are, to zero approximation, zero-point fluctuations of the gluonic fields. One can include also perturbative corrections, order by order. The probability of a non-perturbative, or barrier transition is generically of order

$$\theta_{non-pert} \sim \exp(-const/g^2(a)) \sim (a \cdot \Lambda_{QCD})^\alpha, \quad (1)$$

where  $\alpha > 0$  and can be called a critical exponent.

The terminology is borrowed from theory of percolation (see, e.g.,<sup>3</sup>) and with a good reason. Theory of percolation refers to second order phase transitions. The most interesting case is the so called supercritical phase when there exists a vacuum condensate, or a tachyonic mode. If the phase transition corresponds, in some parameterization to  $\epsilon = 0$  then for small  $\epsilon$  many observables are described in terms of critical exponents. For example, density of condensate is proportional to

$$\theta_{condensate} \sim \epsilon^\alpha, \quad \alpha > 0. \quad (2)$$

In percolation theory, or in the lattice language Eq. (2) refers to the probability of a given link to belong to an infinite cluster. The infinite cluster is a substitution for vacuum condensates of the continuum theory.

Moreover, as is well known, the continuum limit, or  $a \rightarrow 0$  corresponds to the point of second order phase transition. Indeed, the correlation length in lattice units should tend to infinity with  $a \rightarrow 0$ . This is the condition that at physical scale there is no dependence on the lattice spacing. In dimensionless units, the product  $(a \cdot \Lambda_{QCD})$  quantifies closeness to the point of the phase transition.

Next, why the Yang-Mills theories correspond to supercritical phase of the percolation theory. The point is that confinement is due to instability of the perturbative vacuum. The instability is condensation of magnetic degrees of freedom. Thus, we come to expectations (1).

Note that smallness of the probability (1), implies singular non-perturbative fields in same measurements. Indeed, imagine that there is a conserved quantity, expressed in terms of non-Abelian fields. Then constancy of the product of probability (1) and of the field strength implies singular fields. We will have examples later.

## 2.2. Lattice data

In this section we will enumerate various critical exponents, concentrating on definitions and without trying to immediately interpret the data.

- Magnetic strings

The strings are defined as closed, infinitely thin surfaces in the vacuum state which can be open on a 't Hooft line. The 't Hooft line, in turn, is nothing else but trajectory of an external magnetic monopole. Lattice definition of the external monopole is the end point of a Dirac string which carries quantized magnetic flux, determined in terms of the center group.

As is argued in detail in,<sup>4</sup> magnetic strings defined in this way could be identified with the so called center vortices, for review see.<sup>5</sup>

The surfaces percolate through the vacuum. For the probability of a given plaquette to belong to the surfaces one finds:<sup>5,6</sup>

$$\theta_{plaquette} \sim (a \cdot \Lambda_{QCD})^{\alpha_1} , \quad \alpha_1 \approx 2 . \quad (3)$$

Alternatively, Eq (3) can be expressed as statement that the total area of the surfaces is in physical units:

$$(Area)_{strings} \sim \Lambda_{QCD}^2 V_4 , \quad (4)$$

where  $V_4$  is the total volume of the lattice.

One can measure also extra action associated with the strings:<sup>6</sup>

$$S_{strings} \sim (\Lambda_{QCD}^4 V_4)(a \cdot \Lambda_{QCD})^{-\beta_1} \sim \Lambda_{QCD}^2 a^{-2} V_{tot} , \quad \beta_1 \approx 2 . \quad (5)$$

Thus, the non-perturbative fields associated with the strings are singular in the continuum limit of  $a \rightarrow 0$  (as promised above).

To appreciate, which conserved quantity is hiding behind the observation  $\alpha_1 = \beta_1$  (compare (3) and (5)) let us note that, upon fixing  $\alpha_1$  one can derive  $\beta_1 = \alpha_1$  from the requirement that the string tension determined from the stochastic model for the Wilson line is of order  $\sigma \sim \Lambda_{QCD}^2$ .<sup>7</sup>

- Monopole clusters

Monopole trajectories, or 1d defects are defined in specific lattice terms, for review see.<sup>8</sup> Namely, one replaces the original non-Abelian fields by the closest Abelian configuration,  $\{A_\mu^a(x)\} \rightarrow \{A_\mu^3(x)\}$ , where  $a = 3$  is the color index (and we consider SU(2) case). The monopoles are defined then as Dirac monopoles in terms of the projected fields  $\{A_\mu^3(x)\}$ . The physical idea is that confinement is encoded in Abelian degrees of freedom.

The clusters of monopole trajectories possess remarkable scaling properties. In particular, there exists an infinite, or percolating cluster and the probability of a given link to belong to this cluster is of order:<sup>9</sup>

$$\theta_{link}^{perc} \sim (a \cdot \Lambda_{QCD})^{\alpha_2}, \quad \alpha_2 \approx 3. \quad (6)$$

Similar probability in case of finite clusters is of order:

$$\theta_{link}^{finite} \sim (a \cdot \Lambda_{QCD})^{\alpha_3}, \quad \alpha_3 \approx 2. \quad (7)$$

Monopoles are associated with singular non-Abelian fields:<sup>10</sup>

$$S_{monopoles} \sim (L_{mon}^{tot} \cdot \Lambda_{QCD})(a \cdot \Lambda_{QCD})^{-\beta_2}, \quad \beta_2 \approx 1. \quad (8)$$

Monopoles give contribution to the total action similar to that of strings. This is in fact not accidental since monopoles are closely associated with the surfaces, as we will discuss in more detail later.

- Holographic 3d defects

Consider 3d volumes spanned on 2d surfaces, or magnetic strings introduced above. In 4d this volume is not uniquely defined and one concentrates on the minimal volume.<sup>11</sup> The probability of a given elementary cube on the lattice to belong to the volume considered turns to be:

$$\theta_{cube} \sim (a \cdot \Lambda_{QCD})^{\alpha_4}, \quad \alpha_4 \approx 1. \quad (9)$$

This result is important by itself since it means that the surfaces are *not* the so called branched polymers (for discussion and references see<sup>4</sup>).

The 3d defects are closely related to both confinement and spontaneous breaking of chiral symmetry. Existence of such a relation is revealed by a remarkable procedure introduced in Ref.<sup>12</sup> <sup>b</sup> Namely, one strongly modifies links belonging to the 3d volume considered in an algorithmically well defined way:

$$U_\mu^{3d}(x) \rightarrow Z_\mu(x) U_\mu^{3d}(x), \quad (10)$$

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<sup>b</sup>We describe the procedure in version of Ref.<sup>11</sup>

where no summation over  $\mu$  is implied,  $U_\mu(x)$  are original matrices generated with Yang-Mills action, and  $Z_\mu$  are numbers which in  $SU(2)$  case are  $\pm 1$ . Whether  $Z_\mu(x)$  is  $(+1)$  or  $(-1)$  is determined for each particular link in terms of projected fields.<sup>12</sup>

It would take us too far to explain details of determining  $Z_\mu$ . What is important now is that replacement (10) is an ad hoc strong modification of the original gauge fields applied, however, only to a vanishing (in the limit  $a \rightarrow 0$ ) fraction of the total volume.

The result of the substitution (10) is that both confinement and quark condensate,  $\langle \bar{q}q \rangle$  disappear.<sup>12</sup> This happens only if the 3d volume is chosen in the way described above (or any gauge transform of it).

Thus, the whole information on the confinement and chiral symmetry breaking can be encoded in a 3d volume, which is a vanishing fraction of the total 4d volume. Thus, one can talk about a 'holographic' 3d volume.

- Topological defects

As is well known topology of background gluon fields is imprinted on fermionic low-lying modes. Recently, measurements<sup>13</sup> of the volume occupied by topological fermionic modes attracted a lot of attention, for review see.<sup>14</sup> The measurements found non-trivial dependence on the lattice spacing while the expectations—based on the instanton model—were that the topological modes occupy simply the volume of order  $\Lambda_{QCD}^{-4}$ .

In more detail, one considers solutions of the eigenvalue problem

$$D_\mu \gamma_\mu \psi_\lambda = \lambda \psi_\lambda , \quad (11)$$

where the covariant derivative is constructed on the vacuum gluonic fields  $\{A_\mu^a(x)\}$ . For exact zero modes, there is well known index theorem:

$$n_+ - n_- = Q_{top} . \quad (12)$$

Assuming that the topological charge  $Q_{top}$  fluctuates independently by order unit on pieces of 4d volumes measured in physical units:

$$\langle Q_{top}^2 \rangle \sim \Lambda_{QCD}^4 V_{tot} . \quad (13)$$

The so called near-zero modes occupy, roughly speaking the interval

$$0 < \lambda < \pi/L_{latt} , \quad (14)$$

where  $L_{latt}$  is the linear size of the lattice. These modes determine the value of the quark condensate via the Banks-Casher relation:

$$\langle \bar{q}q \rangle = -\pi \rho(\lambda \rightarrow 0) , \quad (15)$$

where  $\lambda \rightarrow 0$  with the total volume tending to infinity.

Measurements<sup>13</sup> with high resolution, that is on the original fields  $\{A_\mu^a(x)\}$  confirmed the general relations (13) and (15). They also demonstrated that the volume occupied by low-lying modes tends to zero in the continuum limit of vanishing lattice spacing,  $a \rightarrow 0$ :

$$\lim_{a \rightarrow 0} V_{mode} \sim (a \cdot \Lambda_{QCD})^{\alpha_5} \rightarrow 0, \quad (16)$$

where numerically  $\alpha_5$  is between 1 and 2. The volume  $V_{mode}$  is defined in terms of the Inverse Participation Ratio (IPR)<sup>c</sup>.

- Stochastic defects

Localization properties were also studied for test color scalar particles in Yang-Mills vacuum.<sup>16</sup> Namely, one studies solutions of the equation

$$D^2 \phi_\lambda = \lambda^2 \phi_\lambda. \quad (17)$$

The analysis has been performed for SU(2) case for color spin  $T=1/2, 1, 3/2$ .

In case of scalars, already in the continuum theory one expects a drastic dependence on the lattice spacing due to the ultraviolet divergent radiative mass correction,  $\delta M^2 = \lambda_{min}^2 \sim \alpha_s a^{-2}$ . To describe particles with physical masses, one uses subtractions. In particular, the subtraction constant  $M^2 = -\lambda_{min}^2$  corresponds to zero renormalized mass. After fixing the renormalized mass, the theory is fully determined perturbatively.

Lattice treatment<sup>16</sup> uncovers new phenomena. There appear localized states which do not propagate at all. These states correspond to eigenvalues in the interval

$$\lambda_{min}^2 < \lambda^2 < \lambda_{mob}^2, \quad (18)$$

where  $\lambda_{mob}^2$  is the so called mobility edge. Probably, the mechanism of the localization in the Yang-Mills case is similar to the Anderson localization, namely, presence of random, or stochastic fields in the vacuum.

If we identify the mobility edge with the radiative mass and introduce subtraction  $M^2 = -\lambda_{mob}^2$ , then there is an advantage that higher eigenvalues  $\lambda_n^2 > \lambda_{mob}^2$  might be associated with standard plane waves. However, the renormalized eigenvalue

$$\tilde{\lambda}^2 \equiv \lambda^2 - M^2$$

is then negative, or tachyonic in the interval (18).

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<sup>c</sup>Independent evidence in favor of shrinking of the regions occupied by topologically non-trivial gluon fields to 3d volumes was obtained in.<sup>15</sup>

More generally, existence of the two invariants,  $\lambda_{mob}^2$  and  $\lambda_{min}^2$  associated with a single particle defies the standard classification scheme of states with respect to the Poincare group.

For the localized states, one introduces new critical exponents:<sup>16</sup>

$$\begin{aligned}\lambda_{mob}^2 - \lambda_{min}^2 &\sim \Lambda_{QCD}^2 (a \cdot \Lambda_{QCD})^{-\beta(T)} , \\ V_{loc}(\lambda_{min}) &\sim \Lambda_{QCD}^{-4} (a \cdot \Lambda_{QCD})^{\alpha(T)} ,\end{aligned}\quad (19)$$

where  $V_{loc}(\lambda_{min})$  is the localization volume.

The exponents  $\alpha(T), \beta(T)$  were measured on the lattice and the measurements brought once again absolutely unexpected results. Namely, the values of  $\alpha(T), \beta(T)$  depend crucially on the color spin:

$$\begin{aligned}\alpha(T=1/2) &\approx 0 , \quad \beta(T=1/2) \approx 0 ; \\ \alpha(T=1) &\approx 2 , \quad \beta(T=1) \approx 1 .\end{aligned}\quad (20)$$

Note that the interval (18) appears to be divergent in the continuum limit for the adjoint case. The renormalization program for the scalar particles in the adjoint representation cannot actually be performed then.<sup>16</sup>

- Shrinking confining string

Most recently, there appeared evidence<sup>17</sup> that the confining string, connecting heavy quarks shrinks to a mathematical line in the continuum limit:

$$\delta^2 \sim \Lambda_{QCD}^{-2} (a \cdot \Lambda_{QCD})^{\alpha_6}, \quad \alpha_6 \approx 2 , \quad (21)$$

where  $\delta$  is the thickness of the flux tube. This is the most challenging observation (compare with the now-standard picture of Ref.<sup>18</sup>)..

### 3. Glimpses of theory

#### 3.1. *Dependence on the resolution of measurements*

An element of theory has already been introduced in fact, in the form of language used. The point is that in the text-book language the lattice spacing is a substitute for an ultraviolet cut off. We are labelling the lattice spacing as resolution of measurements, reserving the use of 'ultraviolet cut off' for the divergences in field theory. There are many observables which can well depend on the resolution of measurements.<sup>19</sup>

Some matrix elements should not depend on the resolution. In terms of critical exponents they correspond to a vanishing index. Consider, for



example, Banks-Casher relation (15). It relates the quark condensate to the density of eigenstates  $\rho(\lambda)$ :

$$\langle \bar{q}q \rangle = \lim_{m \rightarrow 0} m \cdot \int \frac{\rho(\lambda) d\lambda}{(m^2 + \lambda^2)}. \quad (22)$$

With  $a \rightarrow 0$  the number of modes grows,  $\lambda_{max} \sim 1/a$ . However, all modes with  $\lambda \sim 1/a$  drop off from (22) and the only relevant quantity is  $\rho(\lambda \rightarrow 0)$ . Thus, for the quark condensate to be independent on  $a$  we need,

$$\rho(\lambda \rightarrow 0) \sim (a \cdot \lambda_{QCD})^{\gamma_1}, \quad \gamma_1 = 0. \quad (23)$$

which is obeyed by the data.<sup>13</sup> Another example was mentioned above. Namely, by measuring the Wilson line one determines the heavy quark potential,  $V_{\bar{Q}Q}(r)$ . At large  $R$  the potential grows linearly,

$$V_{\bar{Q}Q}(r) \sim \sigma \cdot r, \quad \sigma \sim \Lambda_{QCD}^2 (a \cdot \Lambda_{QCD})^{\gamma_2}, \quad \gamma_2 = 0,$$

and the tension  $\sigma$  (as being directly related to free energy) cannot depend on the lattice spacing. This condition constrains the critical exponents  $\alpha_1, \beta_1$  introduced above,  $\alpha_1 = \beta_1$ .

Note that the Banks-Casher relation does not constrain the volume occupied by the eigenmodes with  $\lambda \rightarrow 0$ . Moreover, there is no other matrix element which would require  $a$ -independence of the volume occupied by the topological fermion modes.<sup>19</sup>

### 3.2. Shrinking topological fermionic modes

In the preceding subsection we emphasized that - as was realized only recently - many observables *could* depend on the resolution. In case of topological fermionic modes it seems possible to make the next step and argue that the topological modes *should* shrink to a vanishing 4d volume.<sup>19</sup>

The essence of the argumentation is that instantons represent a non-unitary contribution to correlator of topological densities<sup>d</sup>. For instanton:

$$\langle G\tilde{G}(x), G\tilde{G}(0) \rangle_{instanton} > 0, \quad x < \rho_{inst}, \quad (24)$$

where  $\rho_{inst}$  is the instanton size. On the other hand, from unitarity alone one can establish<sup>20</sup>

$$\langle G\tilde{G}(x), G\tilde{G}(0) \rangle_{unitary} < 0. \quad (25)$$

<sup>d</sup>Analysis of the unitarity constraints was given much earlier, see<sup>20</sup> and references therein. It is only discussion of implication for the  $a$ -dependence of measurements that is recent.

Eq. (25) means that at any given small  $x$  the perturbative contribution is larger than that of non-trivial topology fluctuations, like instantons.

On the other hand, perturbatively the r.h.s. of Eq (13) is vanishing. The only way to reconcile these, apparently conflicting constraints is to assume for the non-perturbative contribution a singular form:

$$\langle G\tilde{G}(x), G\tilde{G}(0) \rangle_{non-pert} \sim \delta^4(x)\Lambda_{QCD}^4, \quad (26)$$

for checks on the lattice see.<sup>21</sup>

In the language of dispersion relations Eq. (26) implies that the non-perturbative effects are encoded in a subtraction term.<sup>20</sup> In terms of topological fermionic modes Eq (26) implies that the modes are shrinking to a ‘subtraction volume’ which is a vanishing part of the total 4d volume.<sup>19</sup> With a stretch of imagination one can argue that they shrink, most probably, to a 3d volume.<sup>19</sup>

#### 4. Quantum numbers of vacuum defects

##### 4.1. Quantum numbers of 2d defects

The argumentation outlined briefly in the preceding section demonstrates that low-dimensional vacuum defects are in no way a lattice curiosity but could have been predicted within field theory. So far, however, the field-theoretic argumentation was found only in case of topological defects. Thus, we still have to mostly rely on the phenomenology to consider the vacuum defects. We will label the defects with quantum numbers. In this subsection we will consider 2d defects, or lattice strings.

###### *Dimensionality*

The first, and obvious candidate for a quantum number is a dimensionality itself. In particular, for strings it is  $d = 2$ . Note that we do not admit an anomalous fractional dimension of defects. In particular, the surfaces has dimension 2 in physical units, see Eq. (4).

###### *Abelianity*

Non-Abelian fields living on a 2d surface are in fact Abelian. This is not an approximation but a general observation <sup>e</sup>.

Indeed, consider a plane with coordinates  $x_3 = x_4 = 0$ . Then, by non-Abelian field living on this surface one would understand  $G_{12}^a \neq 0$ , where

<sup>e</sup>In all the generality, the observation is made and explored in.<sup>22</sup> Earlier, it was explored in context of more concrete applications, see in particular.<sup>23</sup> Let is also note that the surfaces considered in<sup>22</sup> can be open on monopole lines, with unquantized monopole charges. The surfaces relevant to confinement can be open on the monopole lines with quantized charges, although their own flux is not quantized.

$a$  is the color index. Using gauge invariance one always can rotate this field to a given direction in the color space,

$$G_{12}^a \rightarrow G_{12}^3 ,$$

where, for simplicity of notations we consider SU(2) gauge group.

#### *Chirality*

Thus, the Abelian nature of fields living on a 2d surface looks obvious. There is a subtle further point, however, and it concerns chirality of the gluon fields. The point is that in the 4d Euclidean space the rotation (Lorentz) group is a direct product

$$O(4) = O(3) \times O(3) ,$$

and it is only natural to consider irreducible representation with respect to these groups, which are  $G_{12}^3 \pm G_{34}^3$ . Reserving for an arbitrary combination of the two, we come to the conclusion that 2d surfaces are characterized by two invariants which can be taken  $G^2$  and  $G\tilde{G}$ , none of which is being quantized. In other words, gluon fields living on the surfaces are chiral, generally speaking. It goes without saying that if the surfaces percolate thorough the vacuum, their chirality varies.

#### **4.2. Two-dimensional defects as fundamental ones**

Let us check whether all the data on the critical exponents, which we listed above, can be understood in terms of 2d surfaces. *Monopole clusters* were defined through Abelian projection. At first sight, the Abelian projection has nothing to do with the surfaces. However, now we have learned that surfaces are endowed in fact with Abelian fields. Thus, one could speculate that the surfaces should be seen in the Abelian projection as well (since we should not loose the surfaces by abelianization).

These expectation turn to be true: *within error bars, monopole trajectories lie on the vortices*, see<sup>6</sup> and references therein. Thus monopoles could be manifestations of the same surfaces and just tell us that the surfaces are structured with trajectories.

In the schemes with 2d surfaces as fundamental defects, the *holographic 3d defects* become excessive. Their role is reduced to the role played by their boundaries.

The next question is whether the surfaces carry chirality and explain *topological defects*. In the context of the lattice measurements, the question is whether the topological fermionic modes, with improving resolution shrink to the surfaces.

An attempt to answer this question was undertaken in Ref.<sup>24</sup> through a direct study of correlation between intensities  $\rho_\lambda(x)$  of fermionic modes with eigenvalue  $\lambda$  and of vortices. In more detail, center vortex is a set of plaquettes  $\{D_i\}$  on the dual lattice. Let us denote a set of plaquettes dual to  $\{D_i\}$  by  $\{P_i\}$ . Then the correlator in point is defined as:

$$C_\lambda(P) = \frac{\sum_{P_i} \sum_{x \in P_i} (\rho_\lambda(x) - \langle \rho_\lambda(x) \rangle)}{\sum_{P_i} \sum_{x \in P_i} \langle \rho_\lambda(x) \rangle}. \quad (27)$$

The data does demonstrate strong positive correlation between intensities of topological modes and density of vortices nearby. Moreover, the value of the correlator depends on the eigenvalue and the correlation is strong only for the topological fermionic modes. Although the data does show that the correlator grows for smaller  $a$  it does not allow to fix uniquely the dimensionality of the chiral defects.

Moreover, the percolating strings shed light on the properties of the *stochastic defects*. Indeed, the strings percolate through the vacuum and any percolation is stochastic. Thus, strings are a source of stochastic fields which can be responsible, therefore, for the localization of the test scalar particles.

Most remarkable, the strings bring in a mixed scale,  $\sqrt{\Lambda_{QCD} a^{-1}}$ . In particular, the gluon condensate associated with the string is of order,

$$\langle G^2 \rangle_{strings} \sim \Lambda_{QCD}^2 a^{-2}, \quad (28)$$

see Eq (5). This, mixed scale is manifested also in the properties of the scalar test particles in the adjoint representation, see Eq. (20). Thus, localization of scalar particles might be related to the 2d defects. Identifying the strings with the stochastic component results also in a successful prediction of the confining string tension.<sup>7</sup>

#### 4.3. Fundamental 3d topological defects?

Thus, assumption on the fundamental role of the strings, or 2d defects seems to work very well in many cases. The only remaining challenge is the dimensionality of chiral defects. Namely, measurements of the dimensionality of the topological defects mostly produce the value  $\alpha_5 = 1$  (see Eq (16)) while for surfaces we would have  $\alpha_5 = 2$ . Detailed exposition of evidence in favor of  $\alpha_5 = 1$  can be found in.<sup>14,15</sup>

If indeed  $\alpha_5 = 1$  then the surfaces cannot explain the data on the topological defects and 3d defects should be considered as fundamental as well.

## 5. Limit of classical solutions

In the large  $N_c$ , supersymmetric version of the Yang-Mills theories low-dimensional defects exist as classical solution. In particular, domain wall, or 3d defects carry chirality of the gluon field.<sup>25</sup> One might speculate that quantum numbers of the defects, such as dimensionality and chirality, survive even if the number of colors is not large. Then we would favor 3d topological defects to be fundamental, that is irreducible to the 2d defects.

## 6. Conclusions

Quite recently, it would sound heretic to talk about singular confining fields and non-trivial lattice-spacing dependences of observables. Nowadays, the reality of the lattice measurements is such that it is rather lack of lattice-spacing dependence of an observable that asks for explanation. Indeed, in the text we mentioned the following critical exponents: which specify dependence on the lattice spacing, or resolution:

$$\alpha_1 \approx 1, \alpha_2 \approx 3, \alpha_4 \approx 1, \alpha_5 \approx 2, \alpha_6 \approx 2, \quad (29)$$

$$\alpha(T=1) \approx 2, \beta_1 \approx 1, \beta_2 \approx 1, \beta(T=1) \approx 1.$$

Each of these indices is equal to zero in the standard picture of soft confining fields.

Thus, confining fields, as seen in measurements with high resolution, are certainly singular. What is striking about the indices (29) is that they are –within error bars– integer numbers. Low-dimensional vacuum defects appear to be an adequate language to described the lattice data. The absence of anomalous fractal dimensions is remarkable and we are encouraged to consider dimension of the defects as a quantum number. Further characteristics are Abelianity and chirality of the non-Abelian fields living on the defects.

Absence of anomalous fractal dimensions is natural in case of supersymmetric theories with large number of colors. In this limit vacuum defects exist as classical solutions and supersymmetry provides mechanism for cancellation of quantum corrections. The lattice data (29) might indicate that basic geometric structures survive in case  $N_c = 2, 3$  as well. Thus, it is not ruled out that the data indicate relevance of theory of strings and/or other (fundamental) extended objects to physics of confinement at short distances. If so, one could draw analogies to the developments in particle physics which took place about 50 years ago.

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